Sum and Difference Identities (Part 1)

These notes are intended as a companion to section 7.5 (p. 635 – 640) in your workbook. You should also read the section for more complete explanations and additional examples.

Sum and Difference Identities

Verify that the following statements are true:

a) $\sin(30° + 60°) = \sin 30° \cdot \cos 60° + \cos 30° \cdot \sin 60°$

b) $\cos(30° + 60°) = \cos 30° \cdot \cos 60° - \sin 30° \cdot \sin 60°$

These statements can be generalized to form what are called the **sum and difference identities**.

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

There are also sum and difference identities for the tangent ratio:

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta}$$

Example 2 (sidebar p. 638)
Write each expression in simplest form, then evaluate where possible.

a) $\sin 8x \cdot \cos 3x - \cos 8x \cdot \sin 3x$

b) $$\frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{12}}{1 - \tan\frac{\pi}{6} \cdot \tan\frac{\pi}{12}}$$

Example 3 (sidebar p. 639)
Prove this identity:

$$\sin(\pi - x) = \sin x$$

Example 4 (sidebar p. 640)
Solve the equation $\cos 4x \cdot \cos x + \sin 4x \cdot \sin x = 1$ over the domain $0 \le x < 2\pi$.

Homework: #4, 5, 9, 10ii, 11, 14, 15, 17 in the exercises (p. 641 – 649). Answers on p. 650.